

Materials discussed on 8/11

We have discussed the following.

1. Application of Euler Product

(a) $\sum_{\text{prime } p} p^{-1} = \infty.$

(b) $\zeta(s)^2 = \sum_{n=1}^{\infty} \frac{d(n)}{n^s}$ using product formula

2. Three circle theorem:

If f is analytic on $A(r, R)$ and continuous up to boundary, then for any $r < |z| < R$,

$$\log |f(z)| \leq \frac{\log |z| - \log r}{\log R - \log r} \log M(R) + \frac{\log R - \log |z|}{\log R - \log r} \log M(r)$$

where $M(t) = \sup_{\partial B(t)} |f(z)|.$

3. If $f(0) = 0$ and analytic on $B(1)$, then $\frac{M(r)}{r}$ is non-decreasing on r . In particular, this imply Schwarz Lemma.